Exercise 5

Given $f(x) = x^2 + 2x$ and $g(x) = 6 - x^2$, find $f + g$, $f - g$, fg, and $\frac{f}{g}$. Determine the domain for each function in interval notation.

Solution

Determine each of the functions.

$$f + g = f(x) + g(x) = (x^2 + 2x) + (6 - x^2) = x^2 + 2x + 6 - x^2 = 2x + 6$$

$$f - g = f(x) - g(x) = (x^2 + 2x) - (6 - x^2) = x^2 + 2x - 6 + x^2 = 2x^2 + 2x - 6$$

$$fg = f(x)g(x) = (x^2 + 2x)(6 - x^2) = 6x^2 - x^4 + 12x - 2x^3$$

$$\frac{f}{g} = \frac{f(x)}{g(x)} = \frac{x^2 + 2x}{6 - x^2}$$

The domain of $f + g$, $f - g$, and fg is $(-\infty, \infty)$ because each is a polynomial. The domain of f/g is found by requiring the denominator to not be zero.

$$6 - x^2 \neq 0$$

Solve for x.

$$x^2 \neq 6$$

Take the square root of both sides.

$$\sqrt{x^2} \neq \sqrt{6}$$

Since there's an even power under an even root, and the result is to an odd power, an absolute value sign is needed.

$$|x| \neq \sqrt{6}$$

Remove the absolute value sign by placing $\pm$ on the right side.

$$x \neq \pm\sqrt{6}$$

Therefore, the domain of f/g is $(-\infty, -\sqrt{6}) \cup (-\sqrt{6}, \sqrt{6}) \cup (\sqrt{6}, \infty)$.